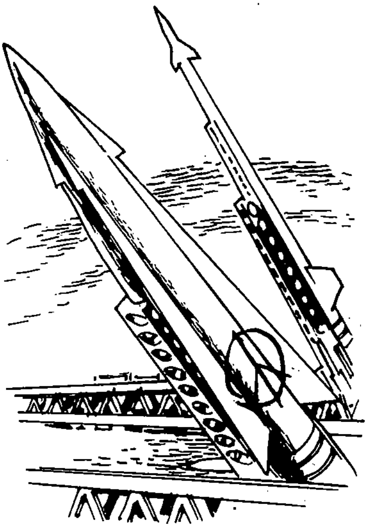

STATISTICS AND PROBABILITY APPLIED TO PROBLEMS OF ANTIAIRCRAFT FIRE IN WORLD WAR II*

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THERE ARE situations in which the statistician is called on to give assistance where exact observational data are so hard to come by that no refined techniques based on mathematical theory can be introduced to solve the problems under investigation. Nevertheless, the statistician's training with its understanding of the meaning of variation and correlation, of randomness and probability, and with its emphasis on the importance of adopting a critical outlook on all assumptions made should help him in handling what at first sight may seem a most intractable problem.

In 1939, at the beginning of World War II, my statistical group from

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University College, London, was attached to the British Ordnance Board. This is an organization of some historical interest, as its origin can be traced back to an appointment made in 1414, the year of the battle of Agincourt. The Board had latterly become involved in certain aspects of the development and acceptance of weapons for the Army, Navy, and Air Force. During the thirties, it had taken the initiative in putting in hand a variety of research projects in matters where information was sadly lacking. One of these investigations, in which the initiative came from Col. A. H. D. Phillips, Superintendent of Applied Ballistics, concerned the problem of the lethality of antiaircraft weapons. The first assignment of my group on joining the Board was to carry on and develop the work already in hand in this field.

THE ANTIAIRCRAFT PROBLEM

First let me try to put the problem into its setting of 30 years ago. As far as the Ordnance Board group was concerned, we had not to consider problems relating to deployment of guns, acquisition of targets, handling of mass attacks, or other important tactical matters. These were questions for the Antiaircraft Command and its Operational Research Section, formed in the summer of 1940. Our work was closely related to the question of weapon design; for example, we had to try to understand more clearly the relationships of the "predictor" that controlled the gun-laying and the setting of the time fuse, the characteristics of the shell and its explosive filling, and the vulnerability of the enemy target to shell fragments. Only then would it be possible to advise what improvements were feasible and likely to be worthwhile.

In this field of research where the terminal action in which we were interested might be taking place several thousand feet above ground, no overall experiment bringing in all the factors concerned was conceivable. Consequently, it was essential to construct a mathematical model of the terminal engagement and then to consider how the parameters of this model might best be estimated. As in so many problems of military or applied industrial science such a model even if simplifying the real situation, as it generally must, serves a necessary purpose. It defines the relationships in the situation, shows how research investigation can be broken into separate pieces and emphasizes at what points lack of sure information is greatest and most hampering. In 1939, the lack of information in the antiaircraft field was great indeed!

Let me first describe the model and the main headings under which gaps in knowledge had to be filled. The problem was one in the field of *probability* because the type of answer we could hope to give was that under certain assumed conditions, there was a probability P that a single shell would destroy (or cripple) the aircraft at which it was fired. As an alternative measure, the reciprocal of P $(1/P)$, the so-called *rounds per bird*, was often quoted. The problem was *statistical* because it was necessary to feed into the model

certain numerical parameter values, derived from the analysis of various forms of observational or experimental data.

The form of the model was conditioned by the fact that, broadly speaking, the factors concerned could be classed under three headings:

(1) *The positioning errors*, that is to say, the distribution of likely positions around the target aircraft at which a fuse-initiated shell would burst.

(2) *The fragmentation characteristics of the shell.* In the problem of heavy antiaircraft guns firing at long range, the chance of a direct hit was small, and therefore damage must be done by fragments from the shell-casing, exploded some distance away. This was, of course, long before the day of guided missiles with homing devices.

(3) *The target vulnerability to shell fragments*, that is to say, the destructive power of fragments, according to their mass and velocity, on hitting the pilot or vital components of the aircraft.

CONSTRUCTION OF THE MATHEMATICAL MODEL

The Positioning Errors. In ground-to-ground firing with artillery shells, there was a good deal of information on the pattern of shell-strikes round the target. This pattern had been found to conform roughly to what is known as the *bivariate normal distribution*, with the density of strikes falling off as we get further from the center of concentration. For this distribution, which is found to represent well the familiar pattern of observed statistical data in studying problems where the error has two sources as here, the contours of equal density are roughly a series of concentric, coaxial ellipses as suggested in Figure 1, the major axis lying along the line joining gun to target, that

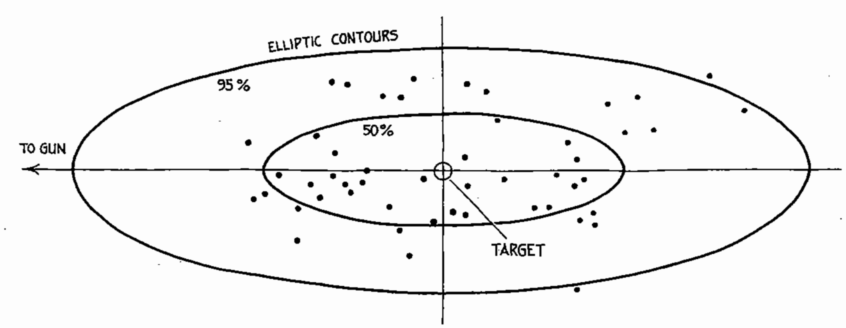

FIGURE 1
Typical chart showing fall of 50 shot

is, the largest errors being in range, not direction.[1] Of course, there might be bias in that what I have termed the center of concentration is not on target, due perhaps to an error in range-finding or a miscalculation of corrections for wind.

It was a reasonable assumption that a similar distribution of errors in position would hold in three dimensions in the sky, although the errors would be much larger in magnitude. However, there was no direct method of examining the error distribution as in the case of the scatter of shell impacts in two dimensions on the ground. It is true that some records were available on the position of shell bursts round a towed target, obtained at antiaircraft practice camps, using a pair of kinetheodolites at each end of a baseline (i.e., theodolites taking continuous film records of angles and time). But these records were not of much value because inevitably the towed target was moving more slowly than a free-flying aircraft and because in 1940 the only personnel whom the Army was prepared to spare for special trials were gunners in course of training.

It would be out of place here to describe the working of the mechanical "predictors" that controlled the firing of the gun, but a number of sources of error were involved in the early stages of the war, largely because it was necessary to have one moving pointer on certain dials followed manually by a second pointer. Later, there was much improvement when a radar element was introduced into the predictor. There was also the error involved in the running of the clockwork time-fuse, even when correctly set. Some useful information on predictor accuracy was obtained in April 1940 from a trial in which a free-flying aircraft, rather than an aircraft-towed target, was followed by several predictor and gun crews simultaneously, and camera recordings of the output dials were synchronised with kinethedolite tracking of the target.

However, when German aircraft began to come over England later in 1940, it was at once clear that the aiming errors under operational conditions were considerably greater than those estimated from trials. We were up against the problem of increased operator inaccuracy under battle stress. The real targets also did not necessarily fly on a straight-line course (as the predictor mechanism assumed) unless on a final bombing run.

All that could be done, therefore, was to assume that under given conditons, the burst of the shell about the target would occur, on repeated firing, in a distribution described mathematically by the three-dimensional analogue of that suggested in Figure 1. The model allowed for the degree of scatter in the direction along the shell trajectory and at right angles to this to be adjusted at will.

[1] The pattern, with only 50 rounds fired, will not of course be regular, but in the diagram it is seen that 26 out of 50 rounds fall within the theoretical 50% ellipse and 48 of 50 within the 95% ellipse.

The Fragmentation Problem. Before the war, the standard trials for determining the fragmentation characteristics of shell were:

(1) Fragmentation in a sandbag "beehive," the shell fragments being recovered, passed successively through various sizes of sieve and (above a certain minimum size) counted and weighed.
(2) Trials to measure the dispersion and penetrating power of fragments by detonating the shell some five feet above ground in a surround of two-inch-thick wooden targets, placed in a semicircle of, say, 30, 60, 90, or 120 feet radius. The detonation was either at rest or obtained from firing the shell (fitted with a percussion fuse) at appropriate velocities against a light bursting-screen sufficient to trigger the shell on impact.

Before refinements were introduced, the number of perforations of the two-inch targets, or of "throughs," as they were termed, per unit area was taken as a comparative index of the damaging power of different types of shell. The target records showed that the main fragment zone of a shell lay between two cones whose axes were that of the shell axis and trajectory at time of burst; in addition there was a small subsidiary nose cone of fragments. If Figure 2 were rotated in a third dimension about the shell axis, it would map out these zones. The greater the velocity of the shell, the further forward the main fragment belt would be thrown.

In addition, analysis of trial data showed that the number of throughs per unit area, that is, 0, 1, 2, . . . , could be well represented by the terms of a well-known probability distribution, the *Poisson distribution,* whose form depended only on a single parameter, the average number of throughs per unit area. For example, if on the *average* there were two throughs per unit area, the Poisson law gives the chances in any one firing of their being 0, 1, 2, 3, 4, 5, 6 throughs as 0.14, 0.27, 0.27, 0.18, 0.09, 0.04, 0.01, ...

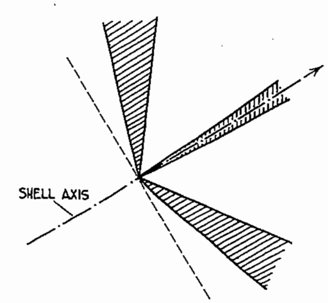

FIGURE 2
Fragment zones

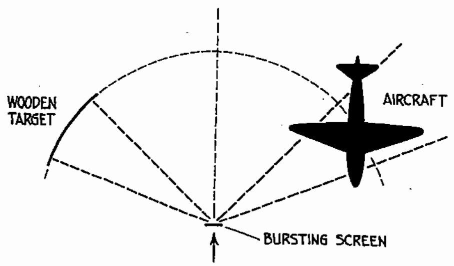

FIGURE 3
Plan of damage trial

Aircraft Vulnerability. In the earliest trials carried out shortly before the war, an aircraft and an arc of large two-inch thick vertical wooden screens were placed beyond and on opposite sides of a small burster-screen at which the shell (with percussion fuse) was fired at a prescribed velocity. Figure 3 illustrates the arrangement. In this way, it was possible to correlate the damage done to the aircraft with the density of throughs in a second, similarly constituted fragment stream. By noting and then painting round the fragment holes after each round was fired, the same targets could be used a large number of times, varying the aspect of attack and the distance of detonation as desired.

It was from the observed correlation of density of throughs and damage to the aircraft that it was possible to introduce into the model calculations a simplified *equivalent vulnerable target*. This was represented in the model by a sphere of a few feet in radius, such that its perforation by at least one lethal fragment (defined as a through) would result in a kill.

THE FUNCTION OF THE COMPLETED MODEL

This simple model, based on the three-dimensional normal distribution (for positioning errors), the Poisson distribution (for perforating fragments), and the equivalent vulnerable target, with bounding surfaces consisting of ellipsoids, cones, and a sphere, was amenable to computation, provided always that meaningful numerical values for the various parameters could be estimated. But the task of filling in these unsurely known elements was far from simple.

We could not hope to get an absolute value of the probability P defined above or of the rounds per bird that would correspond with observed results

against enemy aircraft. In any case, this figure must vary from one type of engagement to another. But we believed that the model, particularly when improved by later refinements based on more sophisticated experiments, would be of value for comparative purposes. It should throw light on what might be achieved by improvement in predictor accuracy, in fuse mechanism, in modification of shell design (for example, by changes in wall thickness and use of more powerful fillings). This concept of constructing a mathematical model, the *changes* in whose end effects (rather than the *absolute* values) can be explored by altering the parameter values, is a basic one in operational research. And it was, of course, the need to introduce a scientific approach into problems of this kind in wartime which led to the postwar demand for more operational-research studies, particularly in industry.

FURTHER REFINEMENTS IN EXPERIMENTS AND MODEL

I have mentioned that the construction of a model and the critical testing of the assumptions on which it has been based, is of value in bringing to light serious gaps in knowledge. In the present case, one of our early puzzles was that when shells were burst in flight within the wooden target surround, the resulting pattern of perforations could not be accurately related to the pattern from a static burst, merely by adding the component forward velocity of the shell. Nor was it easy to link the distribution of fragment sizes from the sandbag collection with the number of perforations of the wood, using any simple assumptions about velocities and retardations. The essential need was for more basic physical experimentation; without this, generalization was impossible.

But such generalization was essential for it was not a practical proposition to ring all the possible changes in trials, of shell design, explosive filling, forward velocity of shell on detonation, and so on. Indeed the ultimate objective must be to predict the characteristics of the fragment distribution for any desired shell velocity from the drawing-board design and a knowledge of the particular explosive filling to be used.

Here we were lucky in getting help from a very skilled scientific team which had been working on explosives in our Safety in Mines Research Establishment at Buxton; these men initiated a program of research that gradually succeeded in disentangling the picture. Shells on which small letters were engraved in successive rings round the circumference were fired at rest, within a surround of strawboard against which small velocity measuring screens were placed. In this way, fragments subsequently collected and weighed could be identified with a particular zone of the shell, and velocities estimated either by direct measurement or, more crudely, from the depth of penetration into the strawboard.

It then became clear that the initial velocity of a fragment varied quite considerably with the part of the casing from which it came and that its size (or weight) also varied with position. To some extent this initial velocity could be related to the dimension of the cross-section of the part of the casing from which the fragment came. With information of this kind it began to be possible to relate the damaging power of a shell to its design and explosive filling.

In another direction the cruder trials, as illustrated in Figure 3 were supplemented by firing individual fragments from high-speed, small-bore guns against selected aircraft components tested in isolation.

Apart from the work in England, some very extensive and informative trials were carried out in the later stages of the war under the direction of a section of the U.S. Navy's Applied Physics Laboratory at Silver Spring, Md., and at its associated proving ground near Albuquerque.

Looking back after a number of years have passed, it seems to me that by 1944 we had really broken the back of the problem. It became possible to make recommendations with some confidence on a number of matters: on the optimum design characteristics of time-fused and proximity-fused shells, on the relative importance of case thickness and explosive filling, on what might be achieved by using methods to control the size of fragments, and on the relative gains to be won by improvement in fire control and in design of shells. Few such questions could have been answered in 1939.

It is a fact, of course, that much of the fundamental research bearing on military problems is only rounded off when it is too late to be of use in the war which provided the stimulus for the effort; and by the next war, the whole conditions of warfare are changed. But I think that the work I have described brought to light a number of principles capable of much wider application.

CONCLUSIONS

The reader of this essay may ask how far the theory and experiment that it has outlined was statistical, rather than contained within the fields of physics and mechanics. It is true that the statistical techniques involved were very elementary, but in order to pull the results together so as to provide comparative figures for the long-run chances of a lethal hit a model with its interpretation involving the theory of probability had to be introduced.

As an interesting corollary, when in the summer of 1944 the American radar proximity fuse was used in the British shell to fire against that ideal antiaircraft target, the V1 flying bomb with its straight-line course, subsequent calculations showed that the expected rounds per kill (rounds per bird) derived from the appropriate probability model, corresponded approximately to the actual operational results.

Finally it seems right to emphasize that in this peculiarly difficult field—the assessment of the operational performance of weapons—the statistician becomes the scientist who must merge his statistical identity into that of a group of men trained in several disciplines, claiming no undue weight for any one of them in the search for answers to the problems in hand.

There are, of course, many other military applications of statistical techniques, particularly in the field of reliability and quality control. In the modern version of the antiaircraft problem we should find much the same general treatment arising in estimating the effectiveness of ground to air guided weapons. With the high-speed computers now available, a much more sophisticated technique of handling the model is possible, but it still must take account of factors under the three headings listed earlier; the positioning and fuse errors, the damaging power of the warhead, and the vulnerability of the target. Although a few high-speed pilotless aircraft may be expended as targets, the number is likely to be far too small to get direct, accurate confirmation of more theoretical estimates of the chances of a kill. In the German V1 Flying Bomb of 1944, our gunners were provided with a standard target that, in one sense at least, cost us nothing!